Lecture 12: Page Replacement

601.418/618 Operating Systems

David Hovemeyer

March 11, 2026

Agenda

- Page replacement
- Page replacement algorithms
- Thrashing

Acknowledgments: These slides are shamelessly adapted from Prof. Ryan Huang's Fall 2022 slides, which in turn are based on Prof. David Mazières's OS lecture notes.

Memory Management

Goals of memory management

- To provide a convenient abstraction for programming
- To allocate scarce memory resources among competing processes to maximize performance with minimal overhead

Mechanisms

- Physical and virtual addressing (1)
- Techniques: Partitioning, paging, segmentation (1)
- Page table management, TLBs, VM tricks (2)

Policies

- **Page replacement algorithms** (3)

Lecture Overview

Review paging and page replacement

Survey page replacement algorithms

Discuss local vs. global replacement

Discuss thrashing

Review: Paging

Recall paging (swapping) from the OS perspective:

- Pages are **evicted to disk when memory is full**
- Pages **loaded from disk when referenced again**
- References to evicted pages cause a TLB miss
  - PTE was invalid, causes fault
- OS allocates a page frame, reads page from disk to the page frame
- OS fills in PTE, marks it valid, and restarts faulting instruction

Dirty vs. clean pages

- Actually, only dirty pages (modified) need to be written to disk
- Clean pages do not
  - but you do need to know where on disk to read them from again!

Paging Challenges

How to resume a process after a fault?

- Need to save state and resume

What to fetch from disk?

- Just needed page or more?

What to evict?

- How to allocate physical pages amongst processes?
- Which of a particular process's pages to keep in memory?
- **A poor choice can lead to horrible performance**
  - cost of paging: disk much, much slower than memory

Page Replacement

When a page fault occurs, the OS loads the faulted page from disk into a page frame of physical memory

At some point, the process used all of the page frames it is allowed to use

- This is likely (much) less than all of available memory

When this happens, the OS must **replace** a page for each page faulted in

- It must evict a page to free up a page frame

The **page replacement algorithm** determines how this is done

- Greatly affect performance of paging (virtual memory)
- Also called page eviction policies

Locality

All paging schemes depend on locality

- Processes reference pages in localized patterns

Temporal locality

- Locations referenced recently likely to be referenced again

Spatial locality

- Locations near recently referenced locations are likely to be referenced soon

While the cost of paging is high, if it is infrequent enough it is acceptable

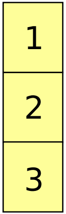
- Processes usually exhibit both kinds of locality during a run, making paging practical

First-In First-Out (FIFO)

Evict oldest fetched page in system

Example reference string 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

3 physical pages:

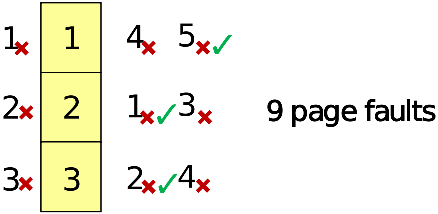


First-In First-Out (FIFO)

Evict oldest fetched page in system

Example reference string 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

3 physical pages: 9 page faults



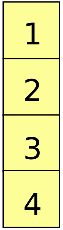
First-In First-Out (FIFO)

Evict oldest fetched page in system

Example—reference string 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

3 physical pages: 9 page faults

4 physical pages:



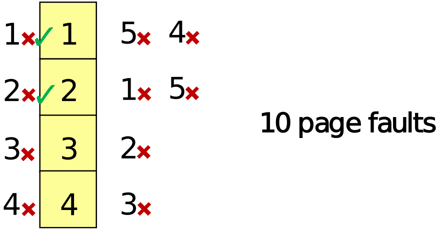
First-In First-Out (FIFO)

Evict oldest fetched page in system

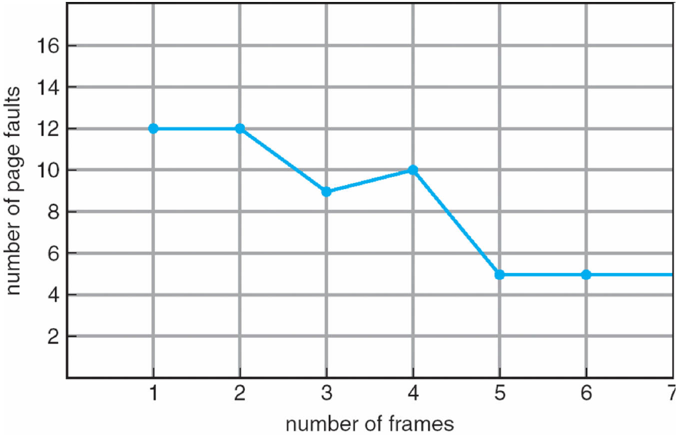
Example—reference string 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

3 physical pages: 9 page faults

4 physical pages: 10 page faults



Belady's Anomaly



More physical memory doesn't always mean fewer faults

Optimal Page Replacement

What is optimal (if you knew the future)?

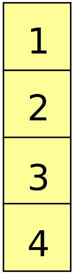
Optimal Page Replacement

What is optimal (if you knew the future)?

- Replace page that will not be used for longest period of time

Example—reference string 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

With 4 physical pages:



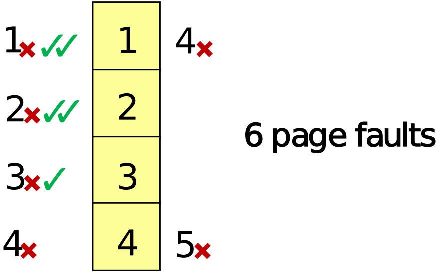
Optimal Page Replacement

What is optimal (if you knew the future)?

- Replace page that will not be used for longest period of time

Example—reference string 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

With 4 physical pages:



Belady's Algorithm

Known as the optimal page replacement algorithm

- Rationale: the best page to evict is the one never touched again
- Never is a long time, so picking the page closest to "never" is the next best thing
- Proved by Belady

Problem: Have to predict the future

Why is Belady's useful then? Use it as a yardstick

- Compare page replacement algorithms with the optimal to gauge room for improvement
- If optimal is not much better, then algorithm is pretty good
- If optimal is much better, then algorithm could use some work
  - Random replacement is often the lower bound

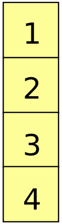
Least Recently Used (LRU)

Approximate optimal with least recently used

- Because past often predicts the future
- On replacement, evict the page that has not been used for the longest time in the **past** (Belady's: **future**)

Example—reference string 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

With 4 physical pages:



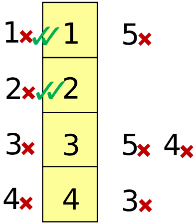
Least Recently Used (LRU)

Approximate optimal with least recently used

- Because past often predicts the future
- On replacement, evict the page that has not been used for the longest time in the **past** (Belady's: **future**)

Example—reference string 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

With 4 physical pages: 8 page faults



Least Recently Used (LRU)

Approximate optimal with least recently used

Example—reference string 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

With 4 physical pages: 8 page faults

Problem 1: **Can be pessimal** – example?

- Looping over memory (then want MRU eviction)

Problem 2: How to implement?

Strawman LRU Implementations

Stamp PTEs with timer value

- E.g., CPU has cycle counter
- Automatically writes value to PTE on each page access
- Scan page table to find oldest counter value = LRU page
- **Problem: Would double memory traffic!**

Keep doubly-linked list of pages

- On access remove page, place at tail of list
- Problem: again, very expensive

What to do?

- Just approximate LRU, don't try to do it exactly

Clock Algorithm

Use accessed bit supported by most hardware

- E.g., Pentium will write 1 to A bit in PTE on first access
- Software managed TLBs like MIPS can do the same

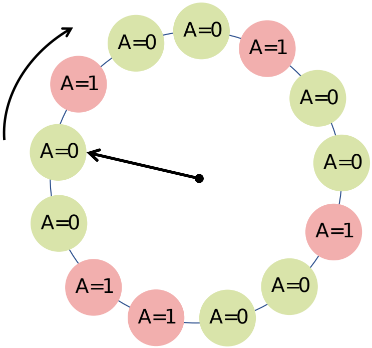
Do FIFO but skip accessed pages

Keep pages in circular FIFO list

Scan:

- page's A bit = 1, set to 0 & skip
- else if A = 0, evict

A.k.a. second-chance replacement



Clock Algorithm

Use accessed bit supported by most hardware

- E.g., Pentium will write 1 to A bit in PTE on first access
- Software managed TLBs like MIPS can do the same

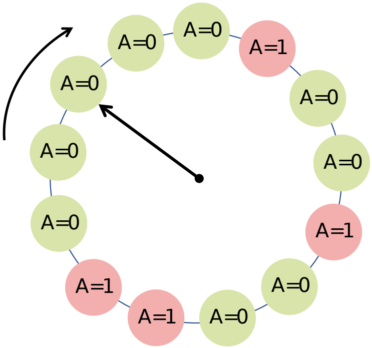
Do FIFO but skip accessed pages

Keep pages in circular FIFO list

Scan:

- page's A bit = 1, set to 0 & skip
- else if A = 0, evict

A.k.a. second-chance replacement



Clock Algorithm

Use accessed bit supported by most hardware

- E.g., Pentium will write 1 to A bit in PTE on first access
- Software managed TLBs like MIPS can do the same

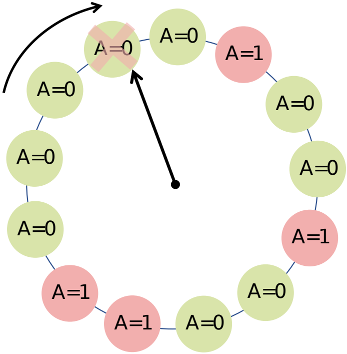
Do FIFO but skip accessed pages

Keep pages in circular FIFO list

Scan:

- page's A bit = 1, set to 0 & skip
- else if A = 0, evict

A.k.a. second-chance replacement



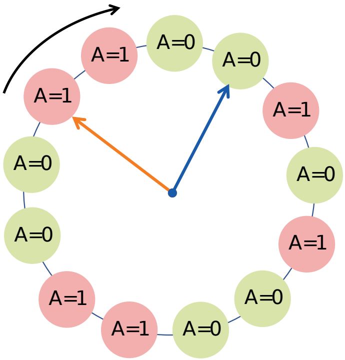
Clock Algorithm (continued)

Large memory may be a problem

- Most pages referenced in long interval

Add a second clock hand

- Two hands move in lockstep
- Leading hand clears A bits
- Trailing hand evicts pages with A=0



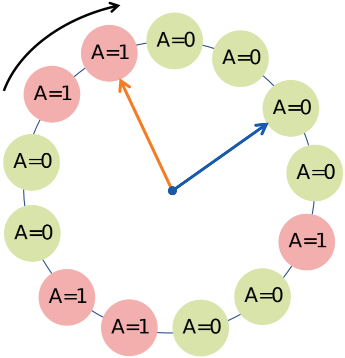
Clock Algorithm (continued)

Large memory may be a problem

- Most pages referenced in long interval

Add a second clock hand

- Two hands move in lockstep
- Leading hand clears A bits
- Trailing hand evicts pages with A=0



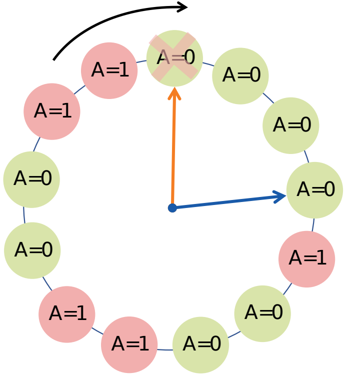
Clock Algorithm (continued)

Large memory may be a problem

- Most pages referenced in long interval

Add a second clock hand

- Two hands move in lockstep
- Leading hand clears A bits
- Trailing hand evicts pages with A=0



Other Replacement Algorithms

Random eviction

- Dirt simple to implement
- Not overly horrible (avoids Belady & pathological cases)

LFU (least frequently used) eviction

- Instead of just A bit, count # times each page accessed
- Least frequently accessed must not be very useful (or maybe was just brought in and is about to be used)
- Decay usage counts over time (for pages that fall out of usage)

MFU (most frequently used) algorithm

- Because page with the smallest count was probably just brought in and has yet to be used

Neither LFU nor MFU used very commonly

Fixed vs. Variable Space

How to determine how much memory to give to each process?

Fixed space algorithms

- Each process is given a limit of pages it can use
- When it reaches the limit, it replaces from its own pages
- **Local replacement**
  - Some processes may do well while others suffer

Variable space algorithms

- Process' set of pages grows and shrinks dynamically
- **Global replacement**
  - One process can ruin it for the rest

Working Set Model

A working set of a process is used to model the dynamic locality of its memory usage

- Defined by Peter Denning in 60s, published at the first SOSP conference

Definition

- $WS(t, w) = \{\text{pages } P \text{ such that } P \text{ was referenced in the time interval } (t, t-w)\}$
- t = time, w = working set window (measured in page refs)

A page is in the working set (WS) only if it was referenced in the last w references

Working Set Size

The working set size is the # of unique pages in the working set

- The number of pages referenced in the interval $(t, t - w)$

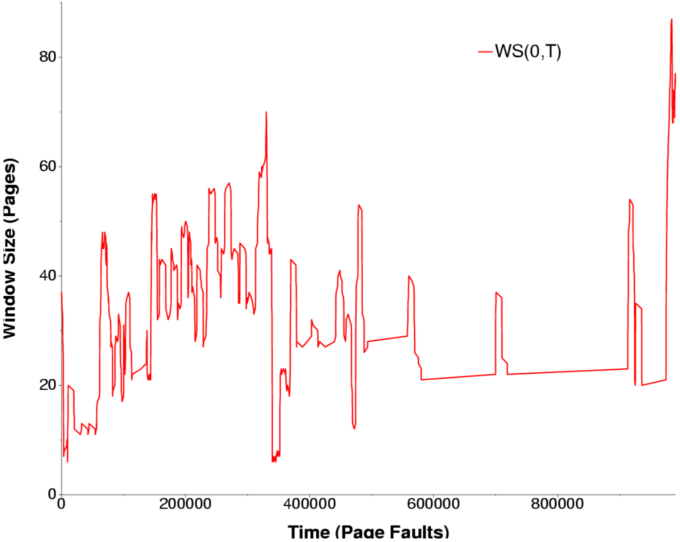
The working set size changes with program locality

- During periods of poor locality, you reference more pages
- Within that period of time, the working set size is larger

Intuitively, want the working set to be the set of pages a process needs in memory to prevent heavy faulting

- Each process has a param w that determines a working set with few faults
- Denning: Don't run a process unless working set is in memory

Example: gcc Working Set



Working Set Problems

Problems

- How do we determine w?
- How do we know when the working set changes?

Too hard to answer

- So, working set is not used in practice as a page replacement algorithm

However, it is still used as an abstraction

- The intuition is still valid
- When people ask, “How much memory does Firefox need?”, they are in effect asking for the size of Firefox’s working set

Page Fault Frequency (PFF)

Page Fault Frequency (PFF) is a variable space algorithm that uses a more ad-hoc approach

- Monitor the fault rate for each process
- If the fault rate is above a high threshold, give it more memory
  - So that it faults less
  - But not always (FIFO, Belady's Anomaly)
- If the fault rate is below a low threshold, take away memory
  - Should fault more
  - But not always

Hard to use PFF to distinguish between changes in locality and changes in size of working set

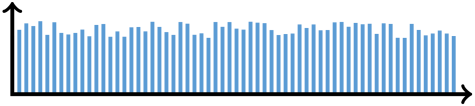
Thrashing

Page replacement algorithms (try to) avoid **thrashing**

- When OS spent most of the time in paging data back and forth from disk
- Little time spent doing useful work (making progress)
- In this situation, the system is **overcommitted**
  - No idea which pages should be in memory to reduce faults
  - Ex: Running Windows95 with 4 MB of memory. . .

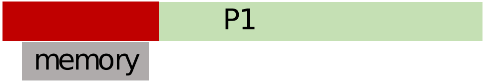
Reasons for Thrashing

Access pattern has no temporal locality (past $\not\approx$ future)

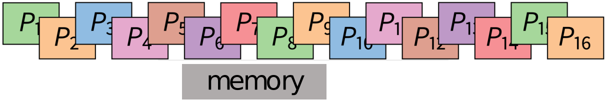


80/20 rule has been violated (usually, 20% of memory gets 80% of accesses)

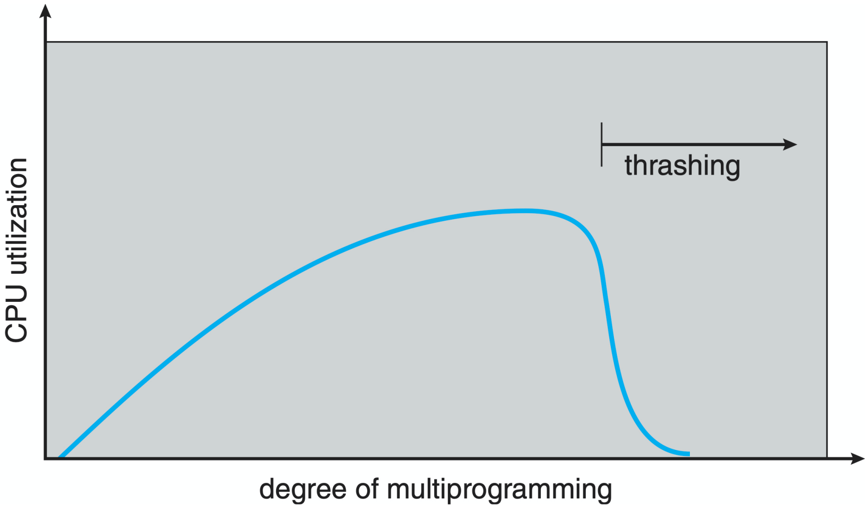
Hot memory does not fit in physical memory



Each process fits individually, but too many for system



Thrashing & Multiprogramming



Dealing with Thrashing

Only run processes if memory requirements can be satisfied

- Thrashing viewed from a caching perspective: given locality of reference, how big a cache does the process need?
- Or: how much memory does the process need in order to make reasonable progress (its working set)

Swapping – write out all pages of a process

Buy more memory. . .

Summary

Page replacement algorithms

- Belady's – optimal replacement (minimum # of faults)
- FIFO – replace page loaded furthest in past
- LRU – replace page referenced furthest in past
  - Approximate using PTE reference bit
- LRU Clock – replace page that is "old enough"
- Working Set – keep the set of pages in memory that has minimal fault rate
- Page Fault Frequency – grow/shrink page set as a function of fault rate

Multiprogramming

- Should a process replace its own page, or that of another?

Next Time

- Dynamic memory allocation